

Quantum mechanics over a q-deformed (0+1)-dimensional superspace

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Abstract

We built up a explicit realization of (0+1)-dimensional q-deformed superspace coordinates as operators on standard superspace. A q-generalization of supersymmetric transformations is obtained, enabling us to introduce scalar superfields and a q-supersymmetric action. We consider a functional integral based on this action. Integration is implemented, at the level of the coordinates and at the level of the fields, as traces over the corresponding representation spaces. Evaluation of these traces lead us to standard functional integrals. The generation of a mass term for the fermion field leads, at this level, to an explicitly broken version of supersymmetric quantum mechanics.

In the last few years the idea that non-commutative geometry could play a central role in the formulation of fundamental physics has attracted some attention[1]. The Connes-Lott version of the Standard Model[2] and Quantum Groups, regarded as endomorphism of some non-commutative space[3], are important examples supporting this idea. In this paper, by means of a simple example, we address the question, how to do quantum mechanics over a non-commutative space?. Among the non-commutative spaces, the quantum plane is an example strongly related to some physical systems. It is clearly understood and, furthermore, the endomorphism of its algebra leads to Quantum Groups[4][3], an structure underlying the integrability of physical models[5]. The quantum plane involves two homogeneously non-commutative "bosonic" coordinates. In this paper we consider an even simpler example well adapted for our purposes. We deal with only one "bosonic" coordinate and two fermionic ones sharing properties with Grassmann variables. As in the quantum plane these coordinates are assumed to be homogeneously non-commutative. We represent the resulting algebra as operators over a standard $(0 + 1)$ -dimensional superspace. Let $t, \eta, \bar{\eta}$ be the coordinates in this standard superspace. The $\bar{\cdot}$ operation can be considered as an involution and we assign a Grassmann number \sharp to each coordinate, i.e.,

$$\begin{aligned} t &\longrightarrow \overline{(t)} = t & \eta &\longrightarrow \overline{(\eta)} = \bar{\eta} \\ \sharp t &= 0 & \sharp \eta &= 1 = -\sharp \bar{\eta} , \end{aligned}$$

the algebraic relations between the coordinates are,

$$\begin{aligned} [t, \eta]^- &\equiv t\eta - \eta t = 0 = [\bar{\eta}, t]^- \equiv \bar{\eta}t - t\bar{\eta} \\ [\bar{\eta}, \eta]^+ &\equiv \bar{\eta}\eta + \eta\bar{\eta} = 0 = [\eta, \eta]^+ = [\bar{\eta}, \bar{\eta}]^+ \end{aligned} \tag{1}$$

Now, let us consider the following operator valued fields on the superspace $\{t, \eta, \bar{\eta}\}$,

$$\begin{aligned} \theta &= \eta e^{-\frac{1}{2}\alpha\bar{\eta}\partial_{\bar{\eta}}} e^{i\alpha t T} = \eta[1 + (e^{-\alpha/2} - 1)\bar{\eta}\partial_{\bar{\eta}}] e^{i\alpha t T} \\ \bar{\theta} &= \sqrt{q} e^{-i\alpha t T} e^{\frac{1}{2}\alpha\eta\partial_{\eta}} \bar{\eta} = \sqrt{q}[1 + (e^{\alpha/2} - 1)\eta\partial_{\eta}] \bar{\eta} e^{-i\alpha t T} \end{aligned} \tag{2}$$

where $\alpha = \ln q$, $q \in \mathbf{R}$ and T is the Hermitian generator of time translations

satisfying $[T, t] = i$. The set $\{t, \theta, \bar{\theta}\}$ has the following commutation relations,

$$\begin{aligned} [t, \theta]_q^- &\equiv t\theta - q\theta t = 0 = [\bar{\theta}, t]_q^- \equiv \bar{\theta}t - qt\bar{\theta} \\ [\bar{\theta}, \theta]_q^+ &\equiv \bar{\theta}\theta + q\theta\bar{\theta} = 0 = [\theta, \theta]^+ = [\bar{\theta}, \bar{\theta}]^+ \end{aligned} \quad (3)$$

Moreover, this algebra is preserved under the q -deformed supersymmetric transformations,

$$\begin{aligned} \delta_\varepsilon t &= -i\bar{\theta}\varepsilon & \delta_{\bar{\varepsilon}} t &= i\bar{\varepsilon}\theta \\ \delta_\varepsilon \theta &= \varepsilon & \delta_{\bar{\varepsilon}} \theta &= 0 \\ \delta_\varepsilon \bar{\theta} &= 0 & \delta_{\bar{\varepsilon}} \bar{\theta} &= \bar{\varepsilon} \end{aligned}, \quad (4)$$

with the parameters ε and $\bar{\varepsilon}$ satisfying,

$$\begin{aligned} [t, \varepsilon]_q^- &= [\bar{\varepsilon}, t]_q^- = 0 \\ [\theta, \varepsilon]_q^+ &= [\bar{\theta}, \bar{\varepsilon}]_q^+ = 0 \\ [\bar{\varepsilon}, \theta]_q^+ &= [\bar{\theta}, \varepsilon]_q^+ = 0 \\ [\bar{\varepsilon}, \varepsilon]_q^+ &= 0 \end{aligned} \quad (5)$$

and the fundamental supersymmetric relation,

$$[\delta_\varepsilon, \delta_{\bar{\varepsilon}}]^- = 2i\varepsilon\bar{\varepsilon}\partial_t. \quad (6)$$

The parameters ε and $\bar{\varepsilon}$ can be expressed also as operators over the super-space $\{t, \eta, \bar{\eta}\}$ as

$$\begin{aligned} \varepsilon &= \zeta e^{-\frac{1}{2}\alpha\bar{\eta}\partial_{\bar{\eta}}} e^{-\alpha\bar{\zeta}\partial_{\bar{\zeta}}} e^{i\alpha t T} \\ \bar{\varepsilon} &= q e^{-i\alpha t T} e^{-\alpha\zeta\partial_{\zeta}} e^{\frac{1}{2}\alpha\eta\partial_{\eta}} \bar{\zeta} \end{aligned} \quad (7)$$

As usual, one may now introduce a scalar q -superfield Φ ,

$$\Phi(t) = x(t) + i\theta\psi(t) - i\bar{\psi}(t)\bar{\theta} + \bar{\theta}\theta d(t) \quad (8)$$

The scalar transformation property of this field under (4), induce the following transformation rules for the q -superfield components $\{x(t), \psi(t), \bar{\psi}(t), d(t)\}$

$$\begin{aligned} \delta_q x(t) &= -i\varepsilon\psi(t) + i\bar{\psi}(t)\bar{\varepsilon} \\ \delta_q \psi(t) &= -iq\bar{\varepsilon}\{[T, x(t)]^- - d(t)\} \\ \delta_q \bar{\psi}(t) &= -iq\{[T, x(t)]^- + d(t)\}\varepsilon \\ \delta_q d(t) &= -\frac{i}{q}\{\varepsilon[T, \psi(t)]_q^- + [\bar{\psi}(t), T]_q^-\bar{\varepsilon}\} \end{aligned} \quad (9)$$

There commutation relations between $\{t, \theta, \bar{\theta}\}$ and the q -superfield components $\{x(t), \psi(t), \bar{\psi}(t), d(t)\}$ are non trivial. Regarding $x(t)$ as a power series in the operator t with complex coefficients, and requiring the invariance of the commutation relations under the q -supersymmetric transformations (4), we get,

$$\begin{aligned}
x(t)\theta &= \theta x(qt) & x(t)\bar{\theta} &= \bar{\theta}x(t/q) \\
\psi(t)\theta &= -\theta\psi(qt) & \psi(t)\bar{\theta} &= -q\bar{\theta}\psi(t/q) \\
\bar{\psi}(t)\theta &= -q^{-1}\theta\bar{\psi}(qt) & \bar{\psi}(t)\bar{\theta} &= -\bar{\theta}\bar{\psi}(t/q) \\
x(qt)\psi(t) &= \psi(t)x(t) & x(t)\bar{\psi}(t) &= \bar{\psi}(t)x(qt) \\
d(t)\theta &= \theta d(qt) & d(t)\bar{\theta} &= q^{-1}\bar{\theta}d(t/q) \\
\psi(t)\bar{\psi}(t) &= -q^{-1}\bar{\psi}(qt)\psi(qt) .
\end{aligned} \tag{10}$$

It is worth remarking that the requirement of q -supersymmetric invariance for the relations in the second line of (10) involves objects such as $\delta\psi(qt)$. Such variations should be obtained from the variation of the superfield $\Phi(qt)$ under the transformation $\delta_q(qt) = q\delta_q t = qi(\bar{\varepsilon}\theta - \bar{\theta}\varepsilon)$, for which the time evolution operator is now $T' = T/q$, satisfying $[T', qt] = i$.

The Grassmannian fields $\psi(t)$ and $\bar{\psi}(t)$ can be represented as,

$$\begin{aligned}
\psi(t) &= e^{-i\alpha t T} e^{\frac{1}{2}\alpha\eta\partial_\eta}\chi(t) \\
\bar{\psi}(t) &= q^{-1/2}\bar{\chi}(t)e^{-\frac{1}{2}\alpha\bar{\eta}\partial_{\bar{\eta}}}e^{i\alpha t T}
\end{aligned} \tag{11}$$

where $\chi(t)$ and $\bar{\chi}(t)$ are q -Grassmannian fields satisfying the algebra,

$$\begin{aligned}
\chi(t)\bar{\chi}(t) &= -q^{-2}\bar{\chi}(t)\chi(t) \\
\chi(t)^2 &= 0 = \bar{\chi}(t)^2
\end{aligned} \tag{12}$$

and the following (anti)commutation relation with the usual superspace co-

ordinates,

$$\begin{aligned}
[\chi(t), t]^- &= 0 \quad , \quad [\bar{\chi}(t), t]^- = 0 \\
[\chi(t), \eta]^+ &= 0 \quad , \quad [\bar{\chi}(t), \bar{\eta}]^+ = 0 \\
[\chi(t), \partial_\eta]^+ &= 0 \quad , \quad [\bar{\chi}(t), \partial_{\bar{\eta}}]^+ = 0
\end{aligned} \tag{13}$$

Now, we introduce the covariant derivatives,

$$\begin{aligned}
D_\theta &\equiv \partial_\theta - q\bar{\theta}T \\
D_{\bar{\theta}} &\equiv \partial_{\bar{\theta}} - qT\theta
\end{aligned} \tag{14}$$

which allows us to build up a q -supersymmetric invariant Lagrangian,

$$L = \frac{1}{2} D_{\bar{\theta}} \Phi D_\theta \Phi - V(\Phi) \tag{15}$$

where $V(\Phi)$ is some polynomial function.

Defining an action on the superspace $\{t, \eta, \bar{\eta}\}$ requires the analogous of integration of L over t , η and $\bar{\eta}$. For the case of our operator valued Lagrangian this can be achieved by taking traces as a replacement of integration. Therefore we need to have an inner product defined on our function space. For the t dependence we take the usual one of square integrable functions, for the Grassmannian sector we use,

$$\langle f | g \rangle \equiv \int d\eta d\bar{\eta} \overline{f(\eta, \bar{\eta})} g(\eta, \bar{\eta}) \tag{16}$$

where the standard Berezin rules for integration on Grassmannian variables are assumed. Hence we define the action by,

$$S(\Phi) = \{Tr_{\{t, \eta, \bar{\eta}\}}[L(\Phi)]\} = \int_0^T dt \sum_n \langle t, e_n | L(\Phi) | e_n, t \rangle , \tag{17}$$

where $\{e_n\}$ denotes a basis for the space of functions of η and $\bar{\eta}$. If we choose that basis to be,

$$e_0 = \frac{1}{\sqrt{2}}(1 + \bar{\eta}\eta) , e_1 = \eta , e_2 = \bar{\eta} , e_3 = \frac{1}{\sqrt{2}}(1 - \bar{\eta}\eta) , \tag{18}$$

its matrix of inner products is,

$$\langle e_i | e_j \rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad (19)$$

This representation space have negative norm states. However, this pseudo-metric leads to an Hermitian inner product,

$$\langle f | g \rangle^* = \langle g | f \rangle, \quad (20)$$

We employ the following results in the evalation of traces,

$$\begin{aligned} \eta e_0 &= \frac{e_1}{\sqrt{2}} & \bar{\eta} e_0 &= \frac{e_2}{\sqrt{2}} & \partial_\eta e_0 &= \frac{-e_2}{\sqrt{2}} & \partial_{\bar{\eta}} e_0 &= \frac{e_1}{\sqrt{2}} \\ \eta e_1 &= 0 & \bar{\eta} e_1 &= \frac{(e_0 - e_3)}{\sqrt{2}} & \partial_\eta e_1 &= \frac{(e_0 + e_3)}{\sqrt{2}} & \partial_{\bar{\eta}} e_1 &= 0 \\ \eta e_2 &= \frac{(e_3 - e_0)}{\sqrt{2}} & \bar{\eta} e_2 &= 0 & \partial_\eta e_2 &= 0 & \partial_{\bar{\eta}} e_2 &= \frac{(e_3 + e_0)}{\sqrt{2}} \\ \eta e_3 &= \frac{e_1}{\sqrt{2}} & \bar{\eta} e_3 &= \frac{e_2}{\sqrt{2}} & \partial_\eta e_3 &= \frac{e_2}{\sqrt{2}} & \partial_{\bar{\eta}} e_3 &= -\frac{e_1}{\sqrt{2}}. \end{aligned} \quad (21)$$

Some explicit results obtained in this way are, for example,

$$Tr_{\{\eta, \bar{\eta}\}}(I) = Tr_{\{\eta, \bar{\eta}\}}(\theta) = Tr_{\{\eta, \bar{\eta}\}}(\bar{\theta}) = 0, \quad Tr_{\{\eta, \bar{\eta}\}}(\bar{\theta}\theta) = q. \quad (22)$$

The “kinetic” term of the Lagrangian (15) is,

$$\begin{aligned} Tr_{\{t, \eta, \bar{\eta}\}}[D_{\bar{\theta}}\Phi D_{\theta}\Phi] &= Tr_{\{t, \eta, \bar{\eta}\}} \left[\bar{\psi}(t)\psi(t) - \bar{\psi}(t)\bar{\theta}\theta T\psi(t) + q\bar{\psi}(t)\bar{\theta}\theta\psi(t)T \right. \\ &\quad \left. - \bar{\psi}(t)T\bar{\theta}\theta\psi(t) + qT\bar{\psi}(t)\bar{\theta}\theta\psi(t) \right] \\ &\quad + q^2 \int_0^T dt [\dot{x}(t)^2 + d(t)^2]. \end{aligned} \quad (23)$$

The explicit calculation of the traces yields the following results,

$$\begin{aligned} Tr_{\{\eta, \bar{\eta}\}}[\bar{\psi}(t)\psi(t)] &= \frac{2}{\sqrt{q}} \sinh(\alpha/2) \bar{\chi}(t)\chi(t) \\ Tr_{\{\eta, \bar{\eta}\}}[\bar{\psi}(t)\bar{\theta}\theta T\psi(t)] &= q\bar{\chi}(t)T\chi(t) \\ Tr_{\{\eta, \bar{\eta}\}}[\bar{\psi}(t)\bar{\theta}\theta\psi(t)T] &= \bar{\chi}(t)\chi(t)T \\ Tr_{\{\eta, \bar{\eta}\}}[T\bar{\psi}(t)\bar{\theta}\theta\psi(t)] &= T\bar{\chi}(t)\chi(t) \end{aligned} \quad (24)$$

The first trace is of central importance. In the undeformed case, the term $\overline{\psi}(t)\psi(t)$ does not contribute. Thus we get the following free Lagrangian,

$$Tr_{\{t,\eta,\overline{\eta}\}}[\frac{1}{2}D_{\overline{\theta}}\Phi D_{\theta}\Phi] = \int_0^T dt \{ \frac{1}{2}q^2[\dot{x}(t)^2 + d(t)^2] + \overline{\chi}(t)[iq\partial_t + m(q)]\chi(t) \} \quad (25)$$

where the mass parameter $m(q)$ is given by,

$$m(q) = \frac{1}{\sqrt{q}} \sinh\left(\frac{\alpha}{2}\right). \quad (26)$$

We take the potential $V(\Phi)$ to be a power series in Φ ,

$$V(\Phi) = \sum_m v_m \Phi^m. \quad (27)$$

It is easy to see that, because of trace over $\{\eta, \overline{\eta}\}$, the effective contribution of this term amounts to,

$$Tr_{\{t,\eta,\overline{\eta}\}}[V(\Phi)] = \int_0^T dt \{ q \frac{\partial V(x)}{\partial x} d(t) + \frac{\partial^2 V(x)}{\partial x^2} \overline{\chi}(t) \chi(t) \}. \quad (28)$$

Therefore the total action $S[x, \overline{\chi}, \chi]$ is,

$$\begin{aligned} S[x, d, \overline{\chi}, \chi] = & \int_0^T dt \{ \frac{1}{2}q^2[\dot{x}(t)^2 + d(t)^2] - q \frac{\partial V(x)}{\partial x} d(t) \\ & + \overline{\chi}(t)[iq\partial_t + m(q) - \frac{\partial^2 V(x)}{\partial x^2}] \chi(t) \}. \end{aligned} \quad (29)$$

In order to evaluate the contribution of the fermionic sector, it is convenient to express $\chi(t)$ in term of the eigenfunctions of the operator $[iq\partial_t + m(q) - \frac{1}{q}V''(x)]$, i.e.,

$$\begin{aligned} \chi(t) &= \sum_n \chi_n \varphi_n(t), \\ \overline{\chi}(t) &= \sum_n \overline{\chi}_n \overline{\varphi}_n(t), \end{aligned}$$

where,

$$\begin{aligned} \chi_n^2 &= \overline{\chi}_n^2 = 0, \\ \chi_m \overline{\chi}_n + \frac{1}{q} \overline{\chi}_n \chi_m &= 0, \end{aligned} \quad (30)$$

and,

$$[iq\partial_t + m(q) - V''(x)]\varphi_n(t) = \lambda_n\varphi_n(t) , \quad (31)$$

with periodic boundary conditions, imposed by supersymmetry. In fact, requiring periodic boundary conditions for the bosonic component $x(t)$ and its q -supersymmetric variation $\delta x(t)$, given in (9), one may constrain the boundary condition on $\psi(t)$. Then, by means of the representation (11), one gets periodic boundary conditions for $\chi(t)$. Obviously, the same holds for its q -Grassmannian conjugated $\bar{\chi}(t)$. So, the eigenfunctions and eigenvalues are,

$$\varphi_n(t) = C e^{\frac{-i}{q} \int_0^t dt' [\lambda_n - m(q) + V''(x)]} \quad (32)$$

$$\lambda_n = \frac{2n\pi q}{T} + m(q) - \frac{1}{T} \int_0^T V''(x) \quad ; n \in \mathbb{Z}.$$

Therefore we get the following fermionic action,

$$\begin{aligned} S_F[x, \bar{\chi}_n, \chi_n] &= \int_0^T dt \left\{ \bar{\chi}(t) \left[iq\partial_t + m(q) - \frac{\partial^2 V(x)}{\partial x^2} \right] \chi(t) \right\} \\ &= \sum_n \lambda_n \bar{\chi}_n \chi_n . \end{aligned} \quad (33)$$

Since $d(t)$ is an auxiliary variable we can eliminate it using its equation of motion. Thus, the action becomes,

$$S[x, \bar{\chi}_n, \chi_n] = \frac{1}{2} \int_0^T dt \left\{ q^2 \dot{x}(t)^2 - [V'(x)]^2 \right\} + \sum_n \lambda_n \bar{\chi}_n \chi_n . \quad (34)$$

For the evaluation of the fermionic contribution it is convenient to express the q -Grassmannian components $\bar{\chi}_n$ and χ_n in terms of standard Grassmann variables,

$$\begin{aligned} \chi_n &= \sigma_n e^{-\frac{1}{2}\alpha \sum_k [\bar{\sigma}_k \partial_{\bar{\sigma}_k} - \partial_{\sigma_k} \sigma_k]} \\ \bar{\chi}_n &= e^{\frac{1}{2}\alpha \sum_k [\bar{\sigma}_k \partial_{\bar{\sigma}_k} - \partial_{\sigma_k} \sigma_k]} \bar{\sigma}_n \end{aligned} , \quad (35)$$

where, $\sigma_n^2 = \bar{\sigma}_n^2 = 0$ and $\{\sigma_m, \bar{\sigma}_n\} = 0$. The generating functional for this action is,

$$Z = \frac{1}{N} \int Dx \quad e^{\frac{i}{2\hbar} \int_0^T dt \{q^2 \dot{x}(t)^2 - [V'(x)]^2\}} \quad Tr_{\{\sigma, \bar{\sigma}\}} [e^{\frac{i}{\hbar} \sum_n \lambda_n \bar{\chi}_n \chi_n}] . \quad (36)$$

To obtain an effective theory, we “integrate out” the fermionic fields, i.e, we evaluate the fermionic contribution to the generating functional,

$$Z_F = Tr_{\{\sigma, \bar{\sigma}\}} [e^{\frac{i}{\hbar} \sum_n \lambda_n \bar{\chi}_n \chi_n}] . \quad (37)$$

We regularize this expression by letting n running from $-N$ to N . Noting that,

$$\bar{\chi}_n \chi_n = e^\alpha \bar{\sigma}_n \sigma_n ,$$

the trace is now easily evaluated, leading to,

$$Tr_{\{\sigma, \bar{\sigma}\}} [e^{\frac{i}{\hbar} \sum_n \lambda_n \bar{\chi}_n \chi_n}] = (i/\hbar)^{2N+1} e^{(2N+1)\alpha} \lambda_{-N} \dots \lambda_N . \quad (38)$$

Since we are mainly interested in the construction of an effective action, the relevant quantity will be the logarithm of the above expression. Following ref. [8], we get

$$Tr_{\{\sigma, \bar{\sigma}\}} [e^{\frac{i}{\hbar} \sum_n \lambda_n \bar{\chi}_n \chi_n}] = \sinh \left[\frac{i}{2q} \int_0^T (m(q) - V''(x)) dt \right] , \quad (39)$$

leading to a generating functional of the form,

$$Z = Z_+ - Z_- , \quad (40)$$

where,

$$Z_\pm = \frac{1}{N} \int Dx \quad e^{\frac{i}{\hbar} \int_0^T dt \left[\frac{1}{2} q^2 \dot{x}^2(t) - \frac{1}{2} [V'(x)]^2 \pm \frac{1}{2q} m(q) \mp \frac{1}{2q} V''(x) \right]} .$$

As shown in ref. [7] and [8], this effective generating functional corresponds to the Hamiltonian,

$$\widetilde{H}_q = \begin{bmatrix} H_q^+ & 0 \\ 0 & H_q^- \end{bmatrix} , \quad (41)$$

with,

$$\begin{aligned} H_q^\pm &= \frac{1}{2q^2} p^2(t) + \frac{1}{2} [V'(x)]^2 \mp \frac{\hbar}{2q} V''(x) \pm \frac{\hbar}{2q} m(q) \\ &= \frac{1}{2} \left(\frac{\mp i}{q} p(x) + V'(x) \right) \left(\frac{\pm i}{q} p(x) + V'(x) \right) + \frac{\hbar}{2q} m(q) . \end{aligned} \quad (42)$$

Observe that $\left[H_q^+ - \frac{1}{2q}m(q)\right]$ and $\left[H_q^- + \frac{1}{2q}m(q)\right]$ are positive-semidefined operators, hence their minimum eigenvalue is 0. The corresponding eigenfunctions are,

$$\varphi_0^\pm(x) = e^{\mp qV(x)} , \quad (43)$$

such that,

$$H_q^\pm \varphi_0^\pm(x) = \mp \frac{1}{2q}m(q)\varphi_0^\pm(x) . \quad (44)$$

So, we can see that the presence of the mass term shift the energy of the ground state away from zero. Moreover, depending whether $q < 1$, the minimum energy is,

$$E_o = \frac{1}{2q}m(q) , \quad (45)$$

belonging to the eigenstate,

$$\varphi_0 = \begin{bmatrix} e^{-qV(x)} \\ 0 \end{bmatrix} \quad (46)$$

or $q > 1$, with eigenvalue,

$$E_o = -\frac{1}{2q}m(q) \quad (47)$$

and eigenstate

$$\varphi_0 = \begin{bmatrix} e^{-qV(x)} \\ 0 \end{bmatrix} \quad (48)$$

Observe that in any case the energy of the ground state is less than zero, thus supersymmetry is explicitly broken.

We end up with some concluding remarks. Starting from a non commutative set of coordinates we considered symmetry operations on them, we dealt with fields over such coordinates, we represent them on certain functional spaces and we took traces as a replacement of integration. Such a procedure is certainly not unique. The selection of a representation is essential and we have no criteria for such a choice. One possibility is to think of such a selection on the same footing as the selection of certain dynamical system.

Regarding the breaking of supersymmetry, it is important to realize that, although we finally get an explicit breaking, at the starting operatorial level we have the full power of supersymmetry to restrict the possible operatorial Lagrangians we can write, in this respect it is similar to a spontaneous breaking.

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